Supplementary material
Appendix 1: Mathematical proofs

**Proposition A1:** Both Hill’s and Leinster & Cobbold’s diversity can be written in form generalized mean, or formally $D^2$ can be defined as generalized mean of rarity with order 1-q.

**Proof:**

Let denote the rarity of species $i$ by $R_i$ and its relative abundance by $p_i$. Generalized mean of rarity with order $r$ is:

$$M_r(p, R) = \left[ \sum_i p_i R_i^r \right]^{\frac{1}{r}}$$  \hspace{1cm} (eq. A1.)

Substituting $r=1-q$ and $R_i=1/p_i$ we get the Hill-diversity:

$$M_{1-q}\left(\frac{1}{p}, \frac{1}{p}\right) = \left[ \sum_i p_i \left( \frac{1}{p_i} \right)^{1-q} \right]^{\frac{1}{1-q}} = \left[ \sum_i p_i (p_i)^{q-1} \right]^{\frac{1}{1-q}} = qD$$  \hspace{1cm} (eq. A2.)

(It is surprising that this extension of their definition to Hill-diversity was not recognized by Patil & Taillie (1982)).

If $R_i = \frac{1}{\sum_j z_{ij}p_j}$ where $z_{ij}$ is the similarity between species $i$ and $j$, and $r=1-q$, then:

$$M_r(p, R) = \left[ \sum_i p_i \left( \frac{1}{\sum_j z_{ij}p_j} \right)^{1-q} \right]^{\frac{1}{1-q}} = \left[ \sum_i p_i \left( \sum_j z_{ij}p_j \right)^{q-1} \right]^{\frac{1}{1-q}} = qD^z$$  \hspace{1cm} (eq. A3.)

**Proposition A2:** LC gamma diversity is higher than alpha diversity if and only if Ricotta-Szeidl’s entropy is concave.

**Proof:**

According to Jost’s (2007) definition alpha-diversity is calculated by the following way:
\[ q D_\alpha = \left( \frac{\sum_{i=1}^{N} q D_i^{1-q}}{N} \right)^{\frac{1}{1-q}} \]  
(eq. A4.)

where \( q D_i \) = diversity of \( i \)-th subsample, \( N \) = number of subsamples.

Between LC-diversity \( (qD) \) and Ricotta-Szeidl’s \( (qH) \) entropy (Ricotta and Szeidl 2006) there is the following relationship (Leinster and Cobbold 2011):

\[ q H = \frac{1 - q D^{1-q}}{q - 1} \]  
(eq. A5.)

after rearrangement:

\[ q D = \left( (1 - q) q H + 1 \right)^{\frac{1}{1-q}} \]  
(eq. A6.)

that substituting in equation 4:

\[ q D_\alpha = \left( \frac{\sum_{i=1}^{N} (1 - q) q H_i + 1}{N} \right)^{\frac{1}{1-q}} = \left( (1 - q) \frac{\sum_{i=1}^{N} q H_i}{N} + 1 \right)^{\frac{1}{1-q}} = \left( (1 - q) q H_\alpha + 1 \right)^{\frac{1}{1-q}} \]  
(eq. A7.)

where \( q H_\alpha \) is the mean Ricotta-Szeidl entropy of subsamples. Applying equation 6 to gamma diversity we get the following relationship:

\[ q D_\gamma = \left( (1 - q) q H_\gamma + 1 \right)^{\frac{1}{1-q}} \]  
(eq. A8.)

Since same formula has to be used to transform alpha and gamma entropy to diversity, \( q D_\alpha \leq q D_\gamma \)

if and only if \( q H_\alpha \leq q H_\gamma \) and the later inequality holds if the entropy function is concave.
Figure A1: Comparing functional a-diversities calculated from different similarity matrices: linear vs exponential transformation of Gower-dissimilarity
Figure A2: Comparing functional a-diversities calculated from different similarity matrices: linear vs modified exponential transformation of Gower-dissimilarity
Figure A3: Comparing functional a-diversities calculated from different similarity matrices: linear transformation of Gower-dissimilarity vs arithmetic mean of overlaps.
Figure A4: Comparing functional a-diversities calculated from different similarity matrices: linear transformation of Gower-dissimilarity vs geometric mean of overlaps.
Figure A5: Comparing functional a-diversities calculated from different similarity matrices: exponential vs modified exponential transformation of Gower-dissimilarity
Figure A6: Comparing functional a-diversities calculated from different similarity matrices: exponential transformation of Gower-dissimilarity vs arithmetic mean of overlaps
Figure A7: Comparing functional a-diversities calculated from different similarity matrices: exponential transformation of Gower-dissimilarity vs geometric mean of overlaps
Figure A8: Comparing functional a-diversities calculated from different similarity matrices: modified exponential transformation of Gower-dissimilarity vs arithmetic mean of overlaps
Figure A9: Comparing functional a-diversities calculated from different similarity matrices: modified exponential transformation of Gower-dissimilarity vs geometric mean of overlaps
Figure A10: Comparing functional a-diversities calculated from different similarity matrices: arithmetic vs geometric mean of overlaps