Ecography

ECOG-01951
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Supplementary material

Appendix 1: Model with demographic stochasticity.

Demographic stochasticity has been shown to interact with Allee effects to shape the growth of low-density populations (Lande 1998, Potapov and Rajakaruna 2013, Roth and Schreiber 2014); as such, we asked how demographic stochasticity affects the outcomes of our simulations. To incorporate demographic stochasticity into our model:

$$N(t)_{x,y} = N(t-1)_{x,y} \exp\left[r\left(1 - \frac{N(t-1)_{x,y}}{K}\right)\left(\frac{N(t-1)_{x,y} - A_{x,y}}{N(t-1)_{x,y}}\right)\right] + S(t)_{x,y} + L(t)_{x,y}$$

we considered the intrinsic rate of population growth r to be sampled from a normal distribution with $\mu = \bar{r}$ and $\sigma = V/[N(t\text{-}1)_{x,y} + 1]$, where V is the variation in individual fitness per unit time (Lande 1993). We modify the equation of Lande (1993) by adding 1 to the denominator so that, as the Allee effect causes populations to decline to extinction, σ does not approach infinity.

We evaluated the importance of demographic stochasticity by comparison with results from the main simulation set using the model with deterministic population dynamics. All parameter values and model scenarios were identical to those used in the deterministic model, except using the stochastic model we set $\bar{r} = 1$ and $V = 0.2$. We conducted $n = 25$ replicates of each scenario. We found that mean rates of spread were higher using the stochastic model, but general patterns with respect to landscape configuration and Allee parameter range were unchanged (Table A1.1). Consequently, we focus the main text on results from the model without demographic stochasticity.

Table A1.1: Mean ± CV (n = 25) spread rates for landscapes varying in spatial configuration and Allee parameter range, using the model with demographic stochasticity.

	Allee parameter range			
Configuration	**Low**	**Medium**	**High**	**High Var.**
Constant	0.728 ± 0.098	0.543 ± 0.136	0.538 ± 0.221	
Gradient	1.027 ± 0.091	0.857 ± 0.108	0.551 ± 0.099	0.679 ± 0.096
Random	1.003 ± 0.146	0.527 ± 0.096	0.504 ± 0.023	0.744 ± 0.119
Fine	1.049 ± 0.136	0.533 ± 0.044	0.504 ± 0.029	0.774 ± 0.130
Coarse	1.078 ± 0.116	0.544 ± 0.140	0.500 ± 0.001	0.784 ± 0.102

References

Lande, R. 1993. Risks of population extinction from demographic and environmental stochasiticty and random catastrophes. - Am. Nat. 142: 911–927.

Lande, R. 1998. Demographic stochasticity and Allee effect on a scale with isotropic noise. - Oikos 83: 353–358.

Potapov, A. and Rajakaruna, H. 2013. Allee threshold and stochasticity in biological invasions: Colonization time at low propagule pressure. - J. Theor. Biol. 337: 1–14.

Roth, G. and Schreiber, S. 2014. Pushed beyond the brink: Allee effects, environmental stochasticity, and extinction. - J. Biol. Dyn. 8: 187–205.

Appendix 2: Methods for quantitative analyses of variability in spread.

Two kinds of variation from mean spread patterns were assessed quantitatively: 1) between-replicate variability in the overall rate of spread, and 2) temporal fluctuations (within-replicate) of the range boundary. Comparing variability in the overall rate of spread indicates whether landscape configuration or Allee effect range affects how strongly the specific configuration of individual landscapes affect rates of spread. Temporal fluctuations in the range boundary were examined to indicate whether spatial variations in Allee effects can induce deviations from the mean spread rate (i.e. pulses of rapid advance or retreat of the range boundary).

Variability in the overall rate of spread was quantified by taking the coefficient of variation (CV) of spread rates. Here, the spread rate was quantified based on the time required for the range boundary to reach the far edge of the landscape ($x = 100$). Because of differences in the mean spread rate between model scenarios, CV was chosen (rather than SD) to normalize variability to the mean rate of spread. These results are reported in Table 2 of the main text.

Temporal fluctuations in the range boundary were quantified by taking the standard deviation (SD) of the residuals from a regression that removed the trend for the range to expand over time. For the constant, random, fine, and coarse landscapes, the position of the invasion front was regressed against time because the mean spread rate was approximately linear. In the gradient landscapes, spread tended to decelerate as the range boundary approached the far edge of the landscape, so the position of the invasion front was regressed against time + time^2. Modeling spread as quadratic, rather than linear, accounted for the deceleration of spread in gradient landscapes. The SD of residuals from each replicate simulation were averaged within model scenario (landscape configuration × Allee parameter range; $n = 50$). These results are reported in Table 3 in the main text.

Appendix 3: Spread "snapshots" for additional landscape configurations

This supplement contains additional examples of spread "snapshots" of the population spatial distribution through time as the range expands. Each group of plots represents a single realization of a stochastic simulation of range expansion in landscapes having a particular configuration and range of Allee thresholds. Where appropriate, the simulation landscape is also depicted to illustrate how spread responds to specific landscape characteristics.

Figure A3.1: Spread in constant low (A = 2.5) and constant medium (A = 5.0) landscapes.

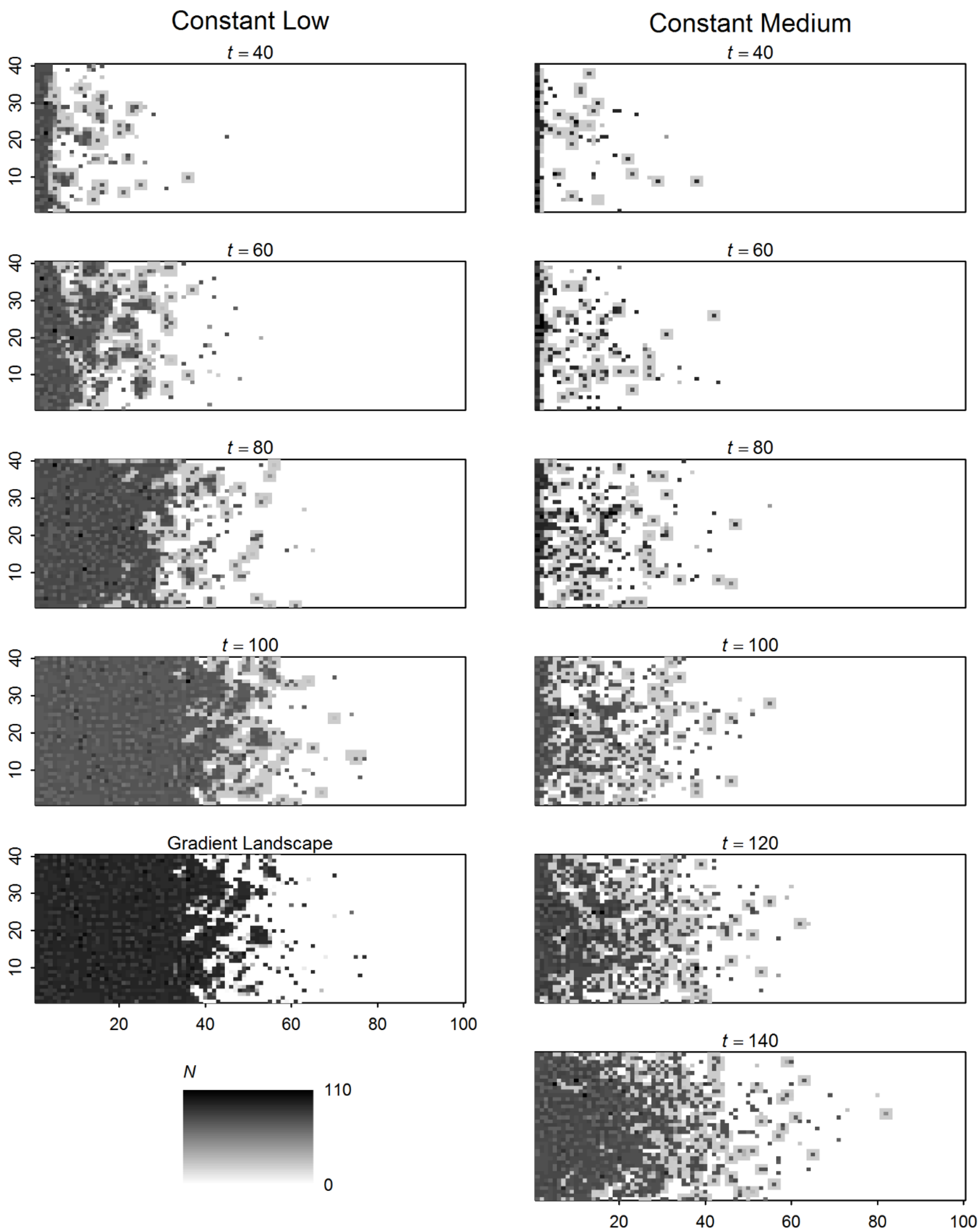

Figure A3.2: Spread in the constant high (A = 7.5) landscape.

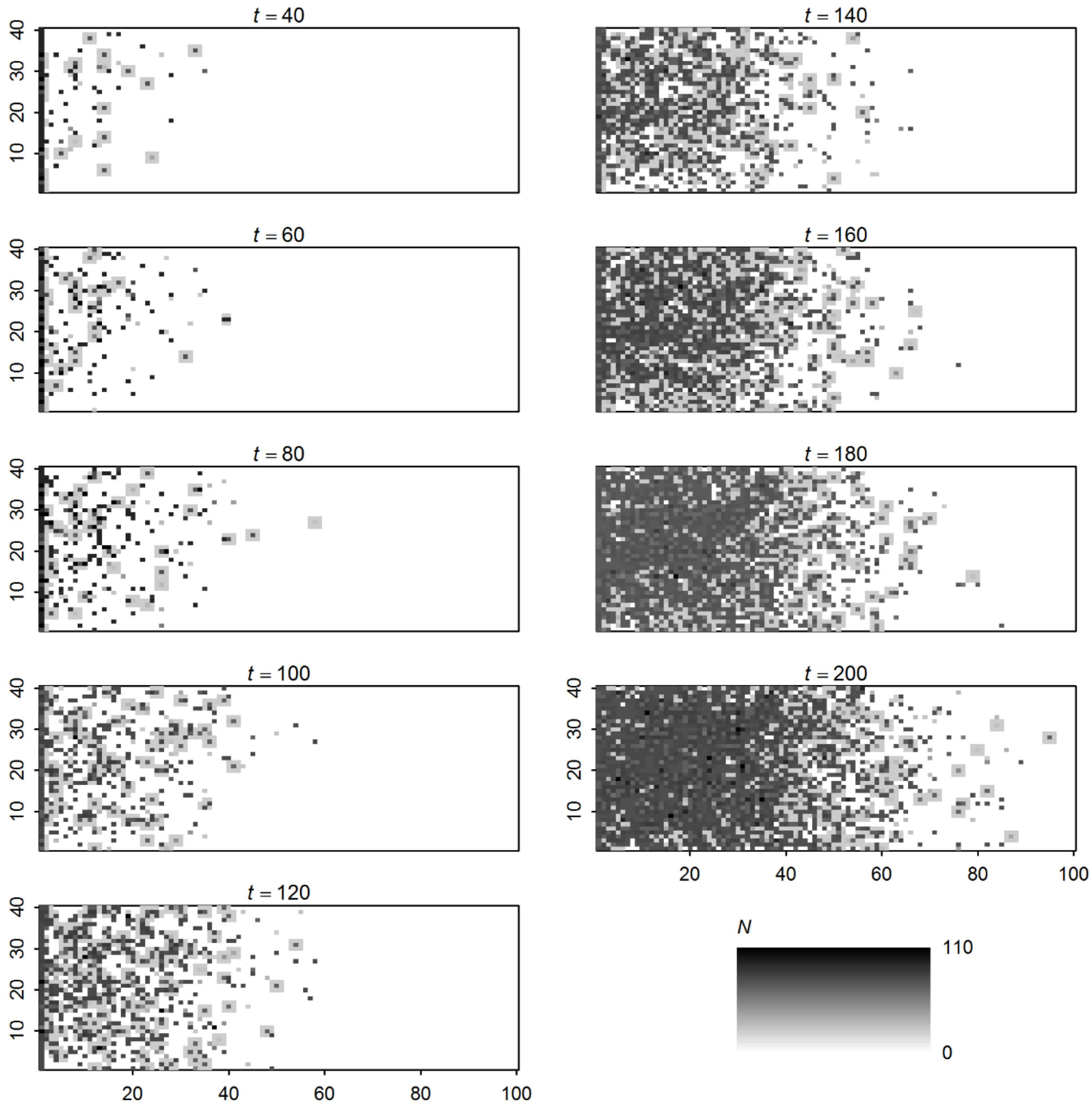

Figure A3.3: Spread in gradient low ($0 \leq A \leq 5$) and gradient medium ($2.5 \leq A \leq 7.5$) landscapes.

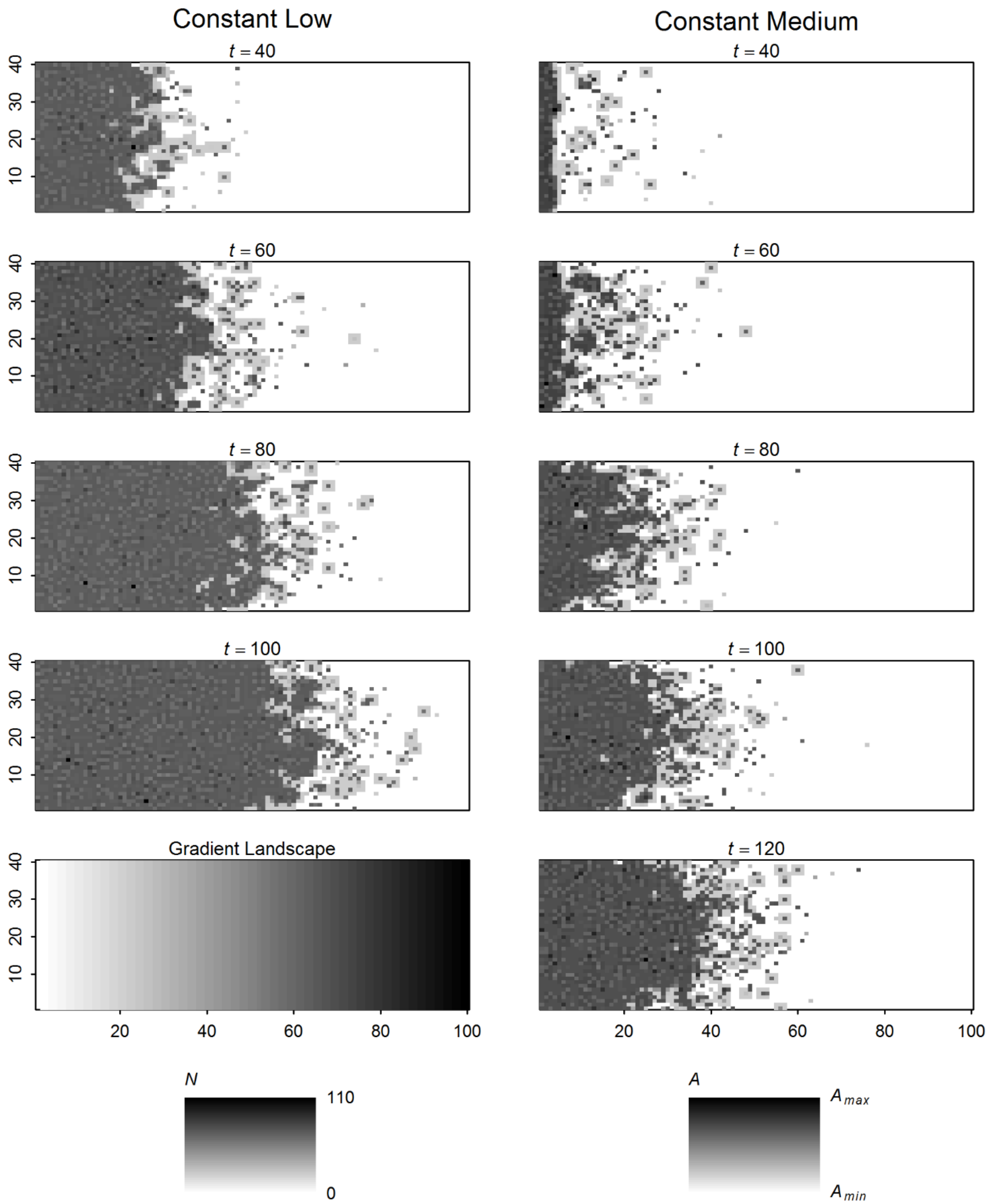

Figure A3.4: Spread in gradient high ($5 \leq A \leq 10$) and gradient high variability ($0 \leq A \leq 10$) landscapes.

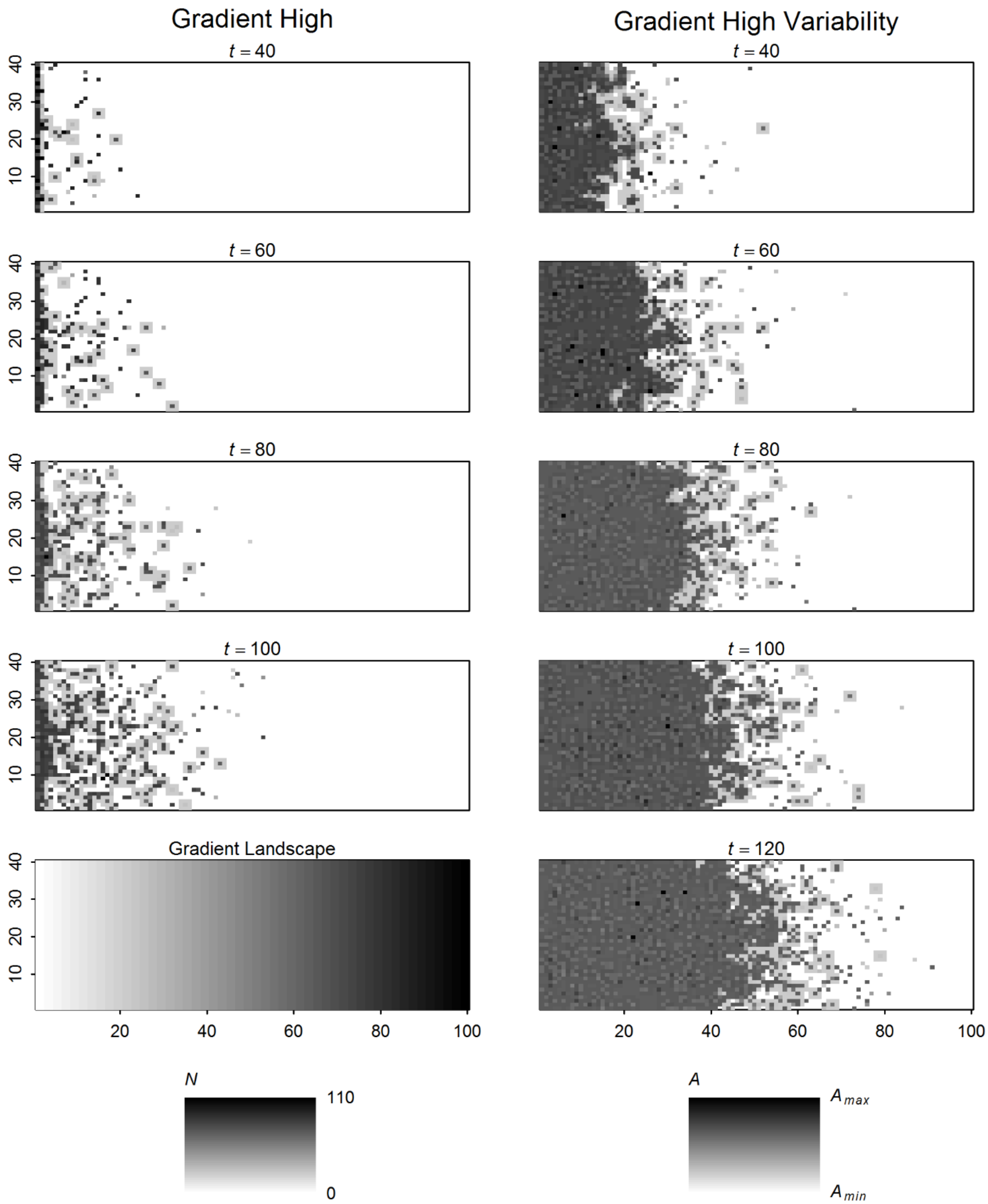

Figure A3.5: Spread in random low ($0 \le A \le 5$) and random medium ($2.5 \le A \le 7.5$) landscapes.

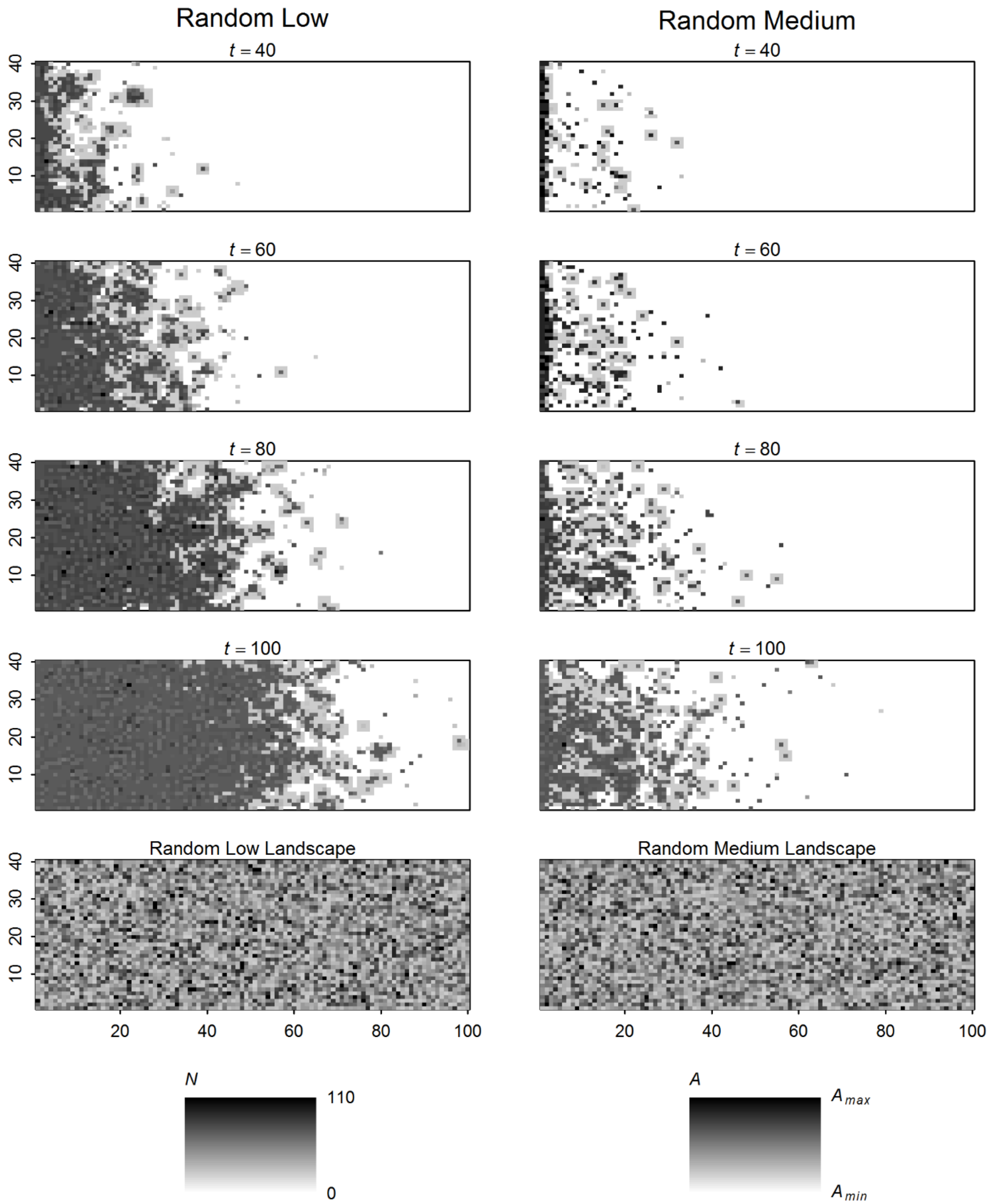

Figure A3.6: Spread in random high ($5 \leq A \leq 10$) and random high variability ($0 \leq A \leq 10$) landscapes.

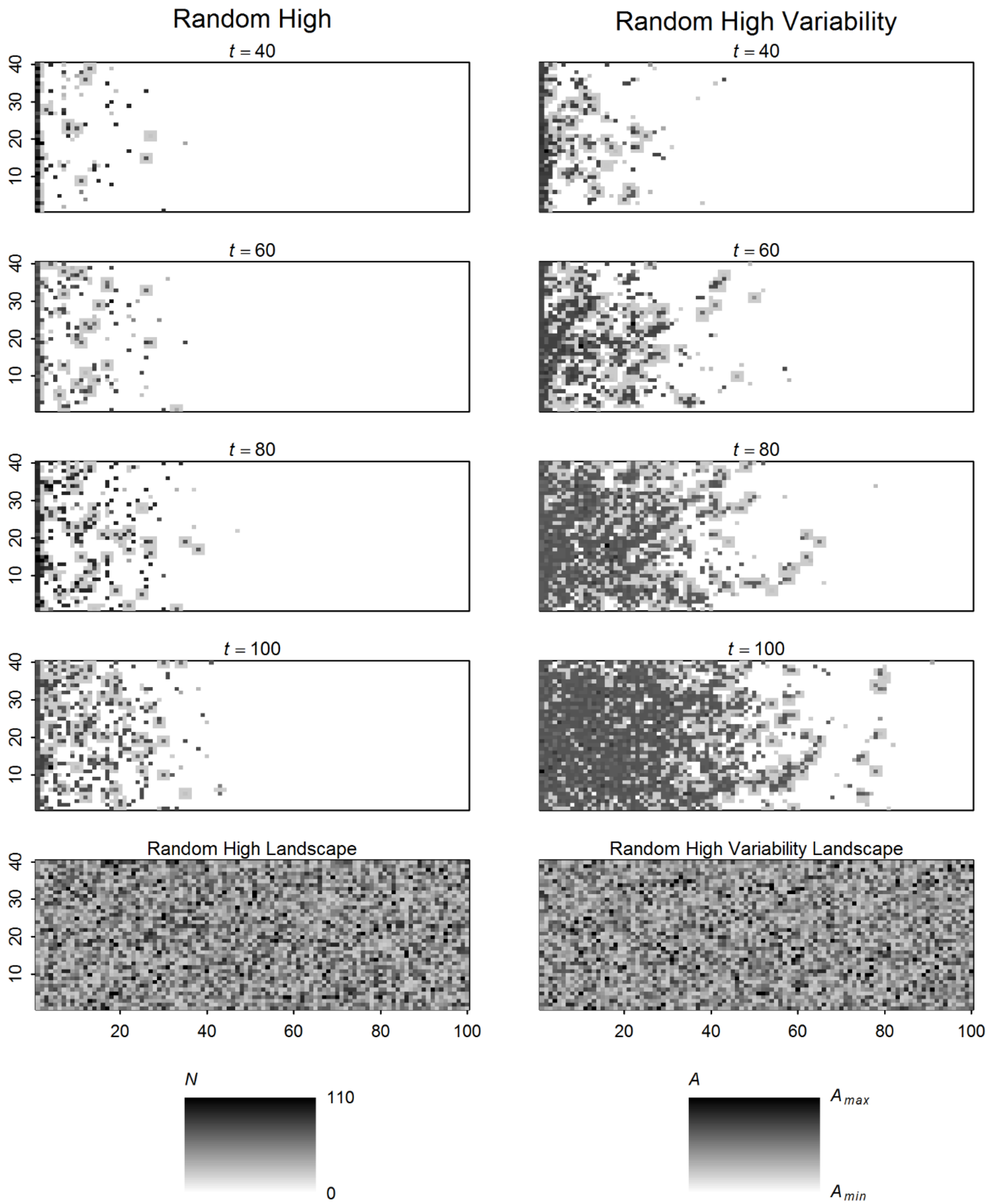

Figure A3.7: Spread in fine low ($0 \leq A \leq 5$) and fine medium ($2.5 \leq A \leq 7.5$) landscapes.

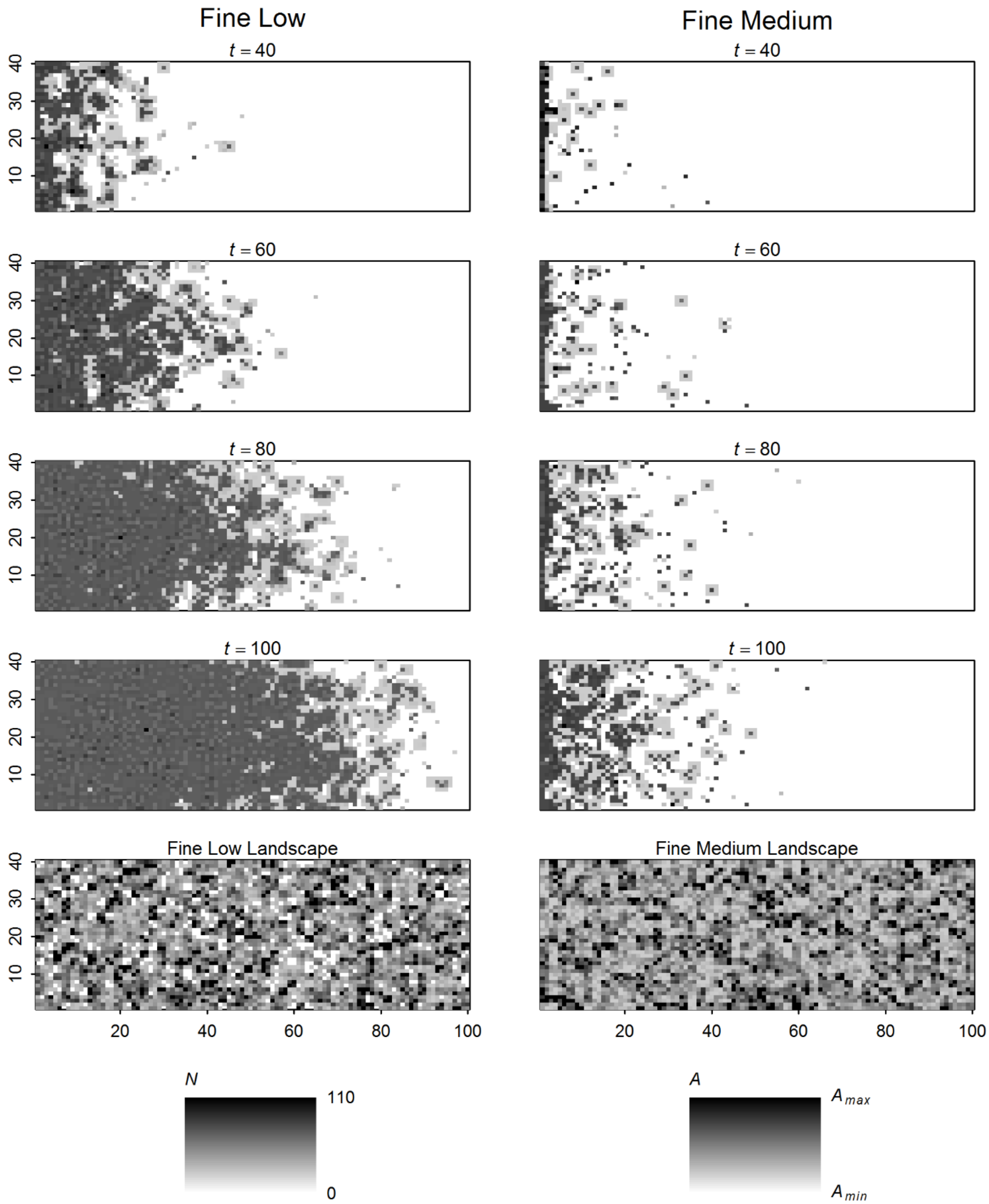

Figure A3.8: Spread in fine high ($5 \leq A \leq 10$) and fine high variability ($0 \leq A \leq 10$) landscapes.

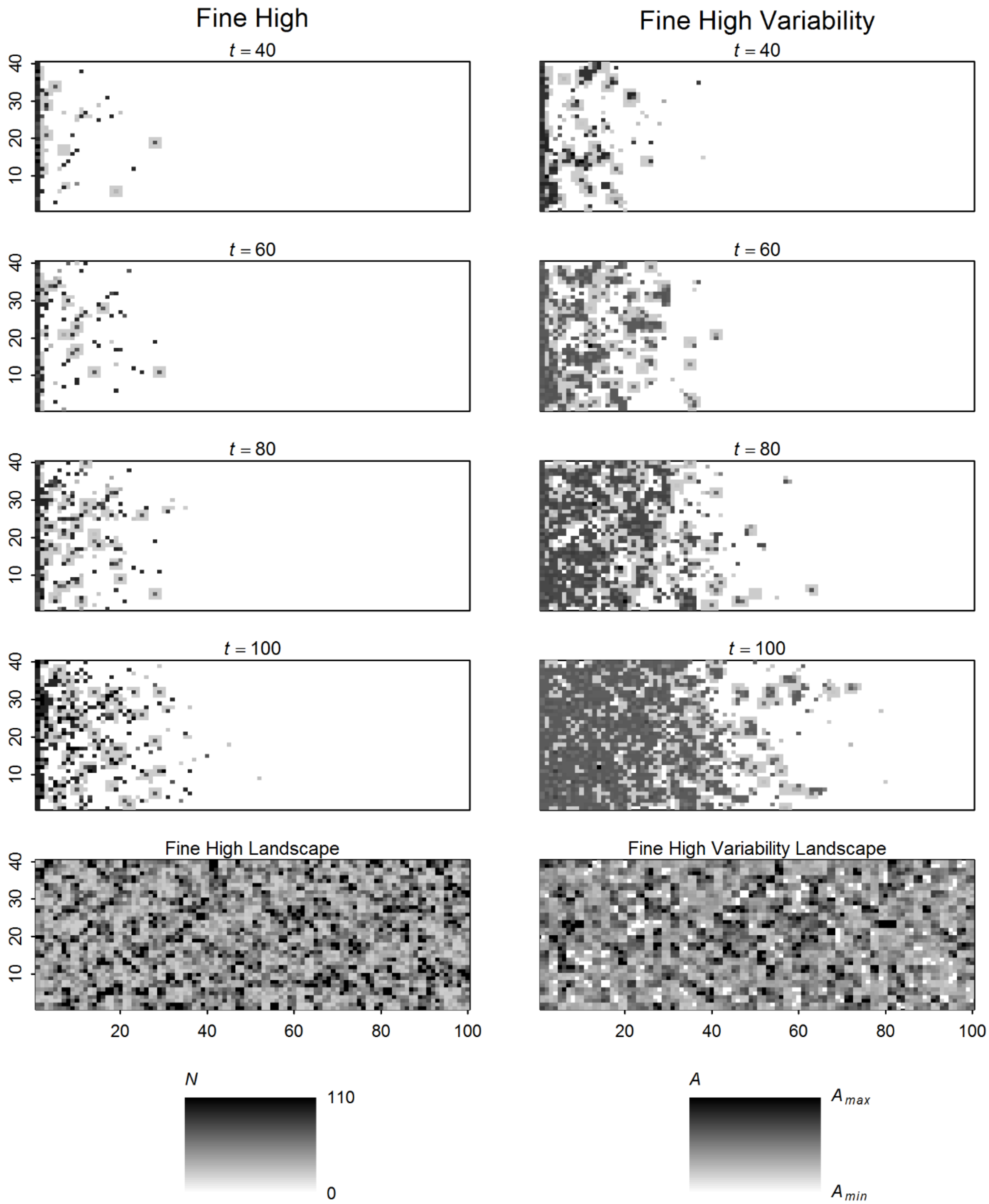

Figure A3.9: Spread in coarse low ($0 \leq A \leq 5$) and coarse medium ($2.5 \leq A \leq 7.5$) landscapes.

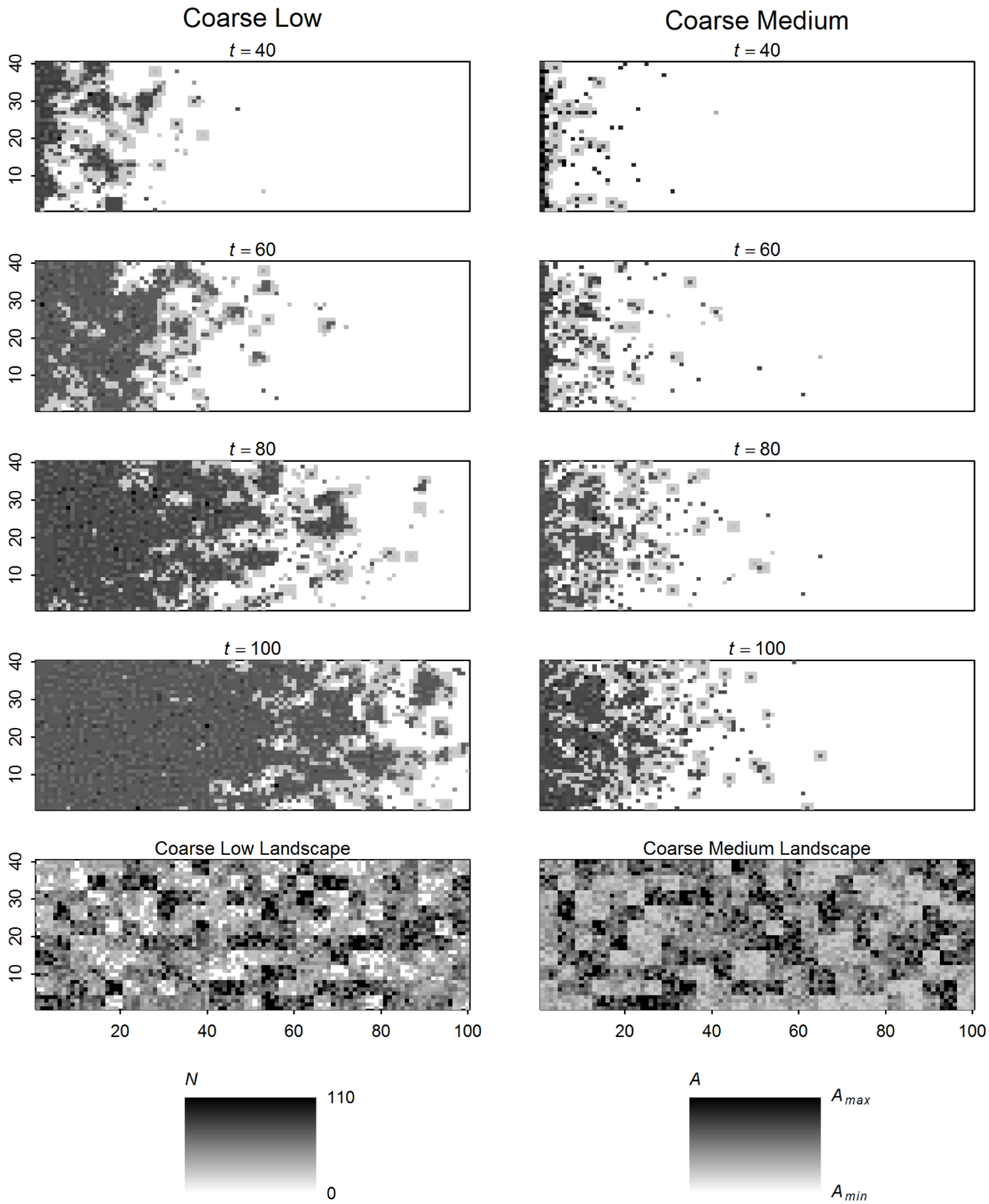

Figure A3.10: Spread in coarse high ($5 \leq A \leq 10$) and coarse high variability ($0 \leq A \leq 10$) landscapes.