Ecography

E6045
Nachman, G. and Borregaard, M. K. 2010. From complex spatial dynamics to simple Markov chain models: do predators and prey leave footprints? – Ecography 33: 137–147.

Supplementary material

Estimation of transition probabilities by means of maximum likelihood

Transition probabilities a_{ij} were estimated by means of maximum likelihood (Hjort and Varin 2007). Assuming a_{ij} to be the same for all plants in state j at time t, the probability that exactly n_{ij} plants of the Q_j plants in state j will change to state i during the next time interval can be found from the multinomial distribution as

$$\mathcal{L}_j = P(n_{1j}, n_{2j}, n_{3j}, n_{4j} \mid Q_j) = \frac{Q_j!}{n_{1j}!n_{2j}!n_{3j}!n_{4j}!} a_{1j}^{n_{ij}} a_{2j}^{n_{2j}} a_{3j}^{n_{3j}} a_{4j}^{n_{4j}} \tag{S1}$$

Hence, the log likelihood function for K observations of transitions from state j becomes

$$\ln \mathcal{L}_j = \sum_{k=1}^{K} \ln Q_{jk}! - \sum_{k=1}^{K}\sum_{i=1}^{4} \ln n_{ijk}! + \sum_{k=1}^{K}\sum_{i=1}^{4} n_{ijk} \ln a_{ijk} \tag{S2}$$

where the index k refers to values of Q_j, n_{ij} and a_{ij} obtained from the kth sample. Since the first two terms of eq. (S1) are constants, the aim is to find values of $a_{ijk} = c_{ij} L(\mathbf{q}_k)^{b_{ij}}$ that maximize the last term of the log likelihood function.

References

Hjort, N. L. and Varin, C. 2007. ML, PL, QL in Markov chain models. – Scand. J. Stat. 35: 64–82.