Supplementary material

Appendix 1. Picture showing site 1 after experimental fire (33 × 33 m). Sites 2 and 3 are same size and characteristics and are located 250 and 300 m apart respectively.
Appendix 2. Maps showing rectangular study regions and point pattern of seedlings emerged after fire.
(A) Site 1, (B) Site 2, (C) Site 3.
(C)
Appendix 3. Combining the data from individual mapped replicate plots into a mean weighted \( O(r) \) function

For statistical analysis it is common to map several replicate plots of a larger point pattern under identical conditions. In this case the resulting second-order statistics of the individual replicate plots can be combined into average second-order statistics (Diggle 2003). This is of particular interest if the number of points in each replicate plot is relatively low. In this case the simulation envelopes of individual analyses would become wide, but combining the data of several replicate plots into average second-order statistics increases the sample size and thus narrows the confidence limits. Average second-order statistics are also an effective way of summarizing the results of several replicate plots. When the patterns are strict replicates of an underlying process, the corresponding estimates \( \hat{K}_i(r) \) of the K-functions from plots \( i \) are identically distributed and a reasonable overall estimate can be obtained by simply averaging the individual K-functions (Diggle 2003: equation 4.20, p. 52). The same is true for the pair-correlation function and the O-ring statistic.

Using the grid-based estimators of Programita and following the notation in Wiegand and Moloney (2004) (their equation 11), the numerical estimator of the bivariate O-ring statistic \( \hat{O}_{ij}(r) \) is calculated as:

\[
\hat{O}_{ij}(r) = \frac{\frac{1}{n_i} \sum_{j=1}^{n_i} \text{Points}_2[R_{ij}^+(r)]}{\frac{1}{n_i} \sum_{j=1}^{n_i} \text{Area}[R_{ij}^+(r)]} \tag{S1}
\]

where \( n_i \) is the number of points of pattern 1, \( R_{ij}^+(r) \) is the ring with radius \( r \) and width \( w \) centered in the \( i \)th point of pattern 1, \( \text{Points}_2[X] \) counts the points of pattern 2 in a region \( X \), and the operator \( \text{Area}[X] \) determines the area of the region \( X \).

To integrate the data of \( M \) different replicates into a single weighted O-ring statistic, the formula for one replicate (eq. S1) is extended by calculating, for each spatial scale \( r \), the average weighted number of points of pattern 2 taken over all \( M \) replicates and the average weighted area taken over all \( M \) replicates:

\[
\hat{O}_{ij}^*(r) = \frac{\frac{1}{N} \left( \frac{1}{n_i} \sum_{j=1}^{n_i} \text{Points}_2[R_{ij}^+(r)] \right) + \ldots + \frac{1}{n_i} \sum_{j=1}^{n_i} \text{Points}_2[R_{ij}^+(r)]}{\frac{1}{N} \left( \frac{1}{n_i} \sum_{j=1}^{n_i} \text{Area}[R_{ij}^+(r)] \right) + \ldots + \sum_{j=1}^{n_i} \text{Area}[R_{ij}^+(r)]} \tag{S2}
\]

where \( i' \) is the \( i \)th point of pattern 1 and replicate \( j \), \( n_{i'} \) is the number of points of pattern 1 and replicate \( j \), and \( N = \sum_i n_{i'} \) is the total number of points of pattern 1 in all replicates. Equation S2 simplifies to:

\[
\hat{O}_{ij}^*(r) = \frac{\sum_{i=1}^{n_i} \text{Points}_2[R_{ij}^+(r)] + \ldots + \sum_{i=1}^{n_i} \text{Points}_2[R_{ij}^+(r)]}{\sum_{i=1}^{n_i} \text{Area}[R_{ij}^+(r)] + \ldots + \sum_{i=1}^{n_i} \text{Area}[R_{ij}^+(r)]} \tag{S3}
\]
The univariate estimators of $O(t)$ is calculated in a manner analogous to the bivariate function by setting pattern 1 equal to pattern 2. In this case, however, the focal points of the circles are not counted:

$$N = \sum_{j=1}^{M} (n_j' - 1).$$  \hspace{1cm} (S4)

References
