

Li, X., Bellard, C., Hu, F. and Li, H. 2019. A comprehensive formula for decomposing change in community similarity into introduction and extinction events. – Ecography doi: 10.1111/ecog.04613

Appendix 1

Derivation of formula for change in similarity based on Sørensen's similarity index

Similar with the derivation in the main text for Jaccard's index, S_{ini} is number of shared species in initial assemblage, S_{cur} is number of shared species in current assemblage, T_{ini} is initial total number of species in the pair, T_{cur} is current total number of species in the pair, $S\sigma r_{ini}$ is initial similarity (Sørensen's index), which is equal to $2 \times S_{ini} / (T_{ini} + S_{ini})$, $S\sigma r_{cur}$ is current similarity, which is equal to $2 \times S_{cur} / (T_{cur} + S_{cur})$. For a given event type, n_i is the number of the given event type i . As mentioned in the main text, $T_{cur} = T_{ini} + n_{I02} + n_{I01} - n_{E20} - n_{E10}$, and $S_{cur} = S_{ini} + n_{I12} + n_{I02} - n_{E21} - n_{E20}$. When the S_{ini} is substituted with $T_{ini} \times J_{ini}$, S_{cur} is also equal to $T_{ini} \times J_{ini} + n_{I12} + n_{I02} - n_{E21} - n_{E20}$. Thus, the change in similarity based on Sørensen's index is as follows:

$$\begin{aligned}
 \Delta S\sigma r &= S\sigma r_{cur} - S\sigma r_{ini} \\
 &= 2 \times S_{cur} / (T_{cur} + S_{cur}) - S\sigma r_{ini} \\
 &= 2 \times (S_{ini} + n_{I12} + n_{I02} - n_{E21} - n_{E20}) / (T_{cur} + S_{cur}) - S\sigma r_{ini} \\
 &= [(T_{ini} + S_{ini}) \times S\sigma r_{ini} + 2 \times (n_{I12} + n_{I02} - n_{E21} - n_{E20})] / (T_{cur} + S_{cur}) - S\sigma r_{ini} \\
 &= [[T_{ini} \times S\sigma r_{ini} + S_{ini} \times S\sigma r_{ini} + 2 \times (n_{I12} + n_{I02} - n_{E21} - n_{E20})] - S\sigma r_{ini} \times (T_{cur} + S_{cur})] / (T_{cur} + S_{cur}) \\
 &= [[T_{ini} \times S\sigma r_{ini} + S_{ini} \times S\sigma r_{ini} + 2 \times (n_{I12} + n_{I02} - n_{E21} - n_{E20})] - S\sigma r_{ini} \times (T_{ini} + n_{I02} + n_{I01} - n_{E20} - n_{E10} \\
 &\quad + S_{ini} + n_{I12} + n_{I02} - n_{E21} - n_{E20})] / (T_{cur} + S_{cur}) \\
 &= [(2 - S\sigma r_{ini}) \times n_{I12} + (2 - 2 \times S\sigma r_{ini}) \times n_{I02} - S\sigma r_{ini} \times n_{I01} - (2 - S\sigma r_{ini}) \times n_{E21} - (2 - 2 \times S\sigma r_{ini}) \times n_{E20} \\
 &\quad + S\sigma r_{ini} \times n_{E10}] / (T_{cur} + S_{cur}) \\
 &= [(2 - S\sigma r_{ini}) \times n_{I12} + (2 - 2 \times S\sigma r_{ini}) \times n_{I02} - S\sigma r_{ini} \times n_{I01} - (2 - S\sigma r_{ini}) \times n_{E21} - (2 - 2 \times S\sigma r_{ini}) \times n_{E20} \\
 &\quad + S\sigma r_{ini} \times n_{E10}] / [(T_{ini} + n_{I02} + n_{I01} - n_{E20} - n_{E10}) + (S_{ini} + n_{I12} + n_{I02} - n_{E21} - n_{E20})] \\
 &= [(2 - S\sigma r_{ini}) \times n_{I12} + (2 - 2 \times S\sigma r_{ini}) \times n_{I02} - S\sigma r_{ini} \times n_{I01} - (2 - S\sigma r_{ini}) \times n_{E21} - (2 - 2 \times S\sigma r_{ini}) \times n_{E20}
 \end{aligned}$$

$$+ S_{\text{ori}} \times n_{\text{E10}}] / (S_{\text{ini}} + n_{\text{I12}} + 2 \times n_{\text{I02}} - n_{\text{E21}} - 2 \times n_{\text{E20}} + T_{\text{ini}} + n_{\text{I01}} - n_{\text{E10}})$$

The formula shows that the magnitude of the effect on similarity of each event type is influenced by initial similarity. The effect magnitudes of I12, I02, E21, and E20 are negatively coupled with initial similarity, while effect magnitudes of I01 and E10 are positively coupled with initial similarity. Similar with Jaccard's index, more events of I12, I02, and E10 have a tendency to lead to homogenization, while more events of I01, E21 and E20 tend to lead to differentiation.