Ecography

ECOG-02009
Botta-Dukát, Z. 2017. The generalized replication principle and the partitioning of functional diversity into independent alpha and beta components. – Ecography doi: 10.1111/ecog.02009

Supplementary material

Appendix 1: Mathematical proofs

Proposition A1: *Both Hill's and Leinster & Cobbold's diversity can be write in form generalized mean, or formally $^qD^Z$ can be defined as generalized mean of rarity with order 1-q.*

Proof:

Let denote the rarity of species *i* by R_i and its relative abundance by p_i. Generalized mean of rarity with order r is:

$$M_r(\boldsymbol{p},\boldsymbol{R}) = \left[\sum_i p_i R_i^r\right]^{\frac{1}{r}} \qquad \text{(eq. A1.)}$$

Substituting $r=1-q$ and $R_i=1/p_i$ we get the Hill-diversity:

$$\mathrm{M}_{1-q}\left(\mathrm{p},\frac{1}{\mathrm{p}}\right) = \left[\sum_{\mathrm{i}} \mathrm{p_i}\left(\frac{1}{\mathrm{p_i}}\right)^{1-\mathrm{q}}\right]^{\frac{1}{1-\mathrm{q}}} = \left[\sum_{\mathrm{i}} \mathrm{p_i}(\mathrm{p_i})^{\mathrm{q}-1}\right]^{\frac{1}{1-\mathrm{q}}} = {}^{\mathrm{q}}\mathrm{D} \qquad \text{(eq. A2.)}$$

(It is surprising that this extension of their definition to Hill-diversity was not recognized by Patil & Taillie (1982)).

If $R_i = {}^{1}/_{\sum_j z_{ij}p_j}$ where zij is the similarity between species i and j, and r=1-q, then:

$$M_r(\boldsymbol{p},\boldsymbol{R}) = \left[\sum_i p_i\left(\frac{1}{\sum_j z_{ij}p_j}\right)^{1-q}\right]^{\frac{1}{1-q}} = \left[\sum_i p_i\left(\sum_j z_{ij}p_j\right)^{q-1}\right]^{\frac{1}{1-q}} = {}^{q}D^{Z} \qquad \text{(eq. A3.)}$$

Proposition A2: *LC gamma diversity is higher than alpha diversity if and only if Ricotta-Szeidl's entropy is concave.*

Proof:

According to Jost's (2007) definition alpha-diversity is calculated by the following way:

$$ {}^{q}D_{\alpha} = \left(\frac{\sum_{i=1}^{N} {}^{q}D_{i}^{1-q}}{N} \right)^{\frac{1}{1-q}} \quad \text{(eq. A4.)} $$

where ${}^{q}D_{i}$ = diversity of *i*-th subsample, *N* = number of subsamples.

Between LC-diversity (${}^{q}D$) and Ricotta-Szeidl's (${}^{q}H$) entropy (Ricotta and Szeidl 2006) there is the following relationship (Leinster and Cobbold 2011):

$$ {}^{q}H = \frac{1 - {}^{q}D^{1-q}}{q-1} \quad \text{(eq. A5.)} $$

after rearrengment:

$$ {}^{q}\mathrm{D} = \left((1-\mathrm{q})\ {}^{q}\mathrm{H} + 1 \right)^{\frac{1}{1-q}} \quad \text{(eq. A6.)} $$

that substituting in equation 4:

$$ {}^{q}\mathrm{D}_{\alpha} = \left(\frac{\sum_{i=1}^{N} (1-\mathrm{q})\ {}^{q}\mathrm{H} + 1}{\mathrm{N}} \right)^{\frac{1}{1-q}} = \left(\frac{(1-\mathrm{q}) \sum_{i=1}^{N} {}^{q}\mathrm{H}_{i}}{\mathrm{N}} + 1 \right)^{\frac{1}{1-q}} = \left((1-\mathrm{q})\ {}^{q}\mathrm{H}_{\alpha} + 1 \right)^{\frac{1}{1-q}} \quad \text{(eq. A7.)} $$

where ${}^{q}H_{\alpha}$ is the mean Ricotta-Szeidl entropy of subsamples. Applying equation 6 to gamma diversity we get the following relationship:

$$ {}^{q}\mathrm{D}_{\gamma} = \left((1-\mathrm{q})\ {}^{q}\mathrm{H}_{\gamma} + 1 \right)^{\frac{1}{1-q}} \quad \text{(eq. A8.)} $$

Since same formula has to be used to transform alpha and gamma entropy to diversity, ${}^{q}D_{\alpha} \leq {}^{q}D_{\gamma}$ if and only if ${}^{q}H_{\alpha} \leq {}^{q}H_{\gamma}$ and the later inequality holds if the entropy function is concave.

Appendix 2: Supplementary Figures

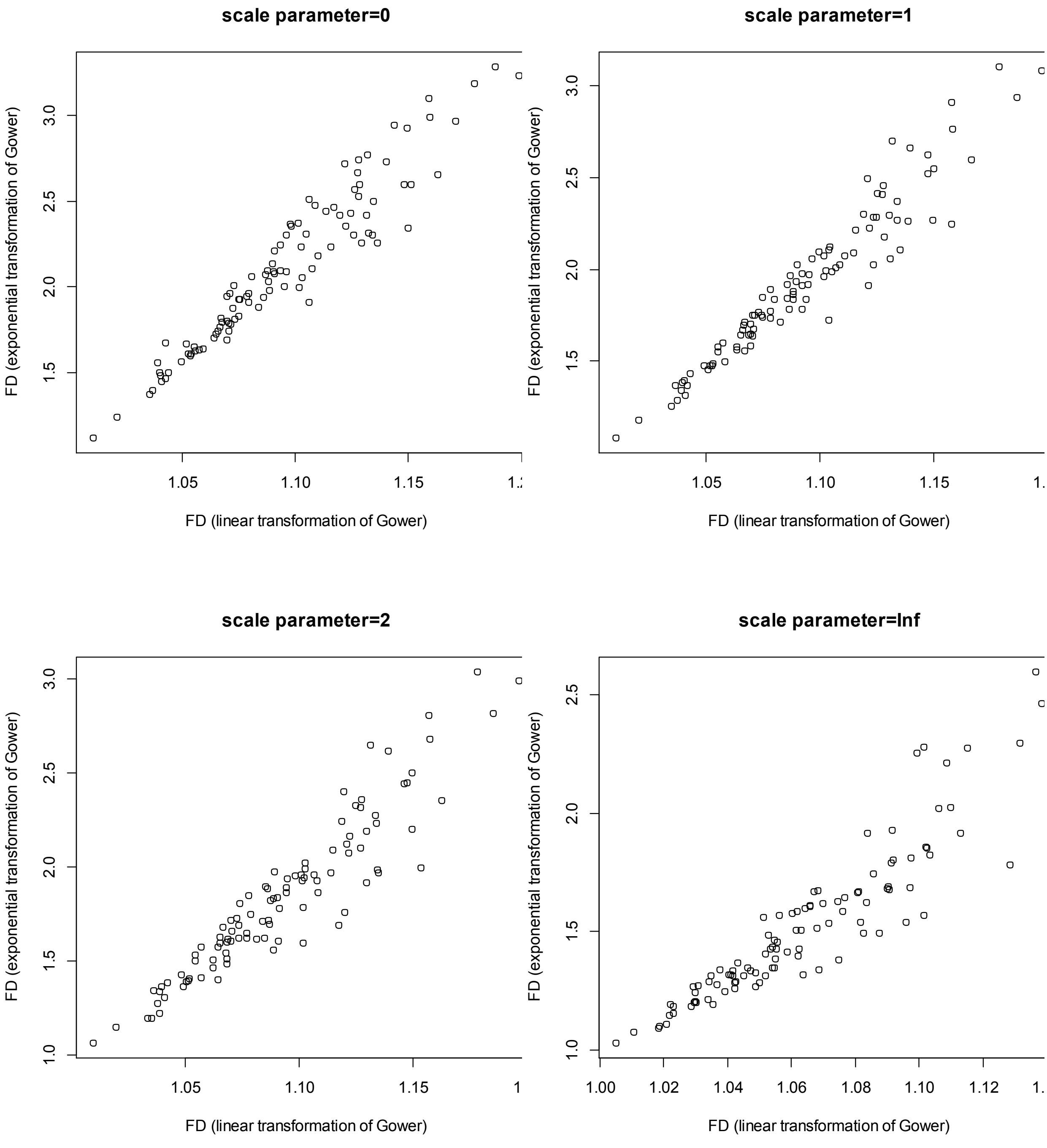

Figure A1: Comparing functional a-diversities calculated from different similarity matrices: linear vs exponential transformation of Gower-dissimilarity

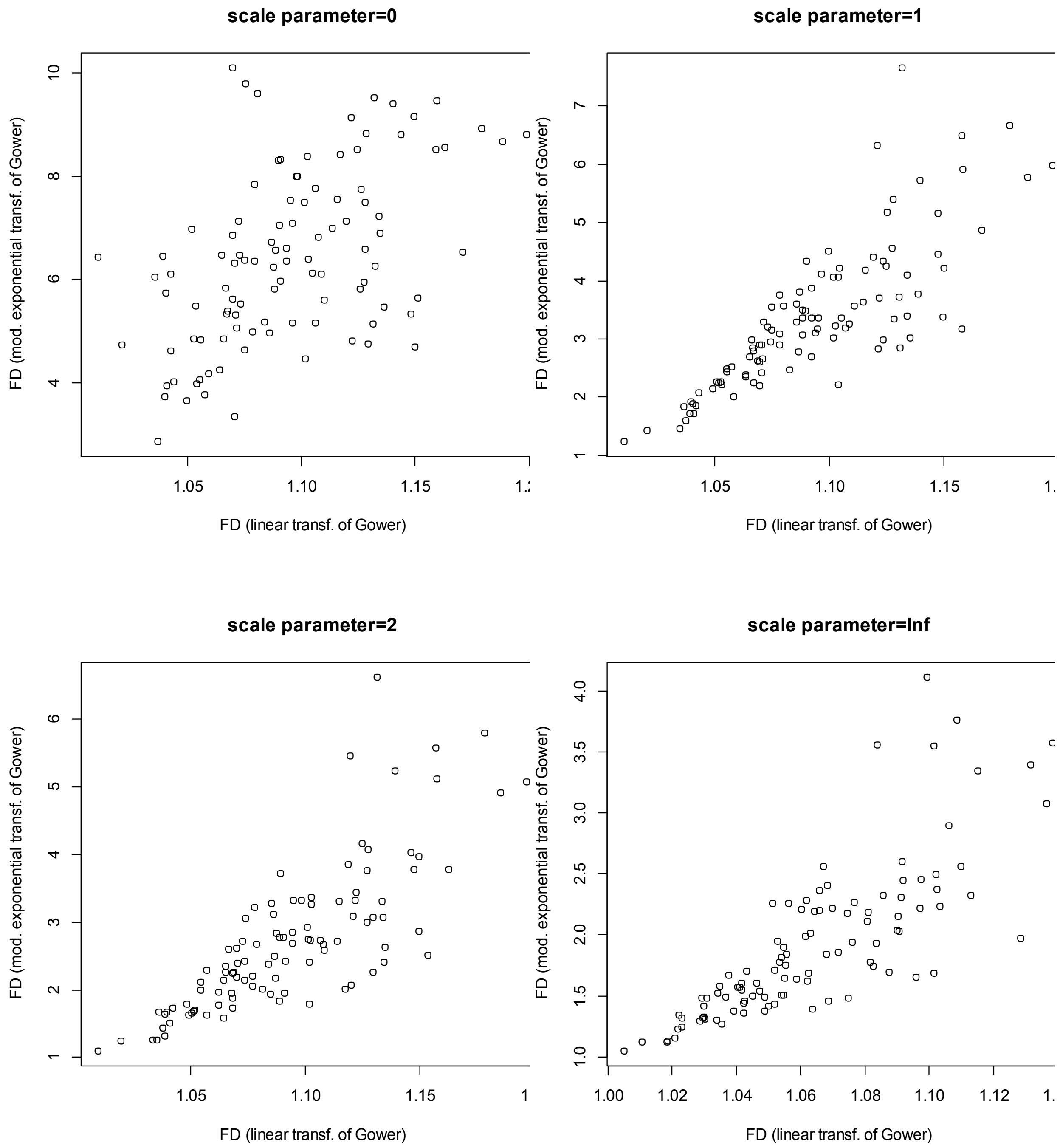

Figure A2: Comparing functional a-diversities calculated from different similarity matrices: linear vs modified exponential transformation of Gower-dissimilarity

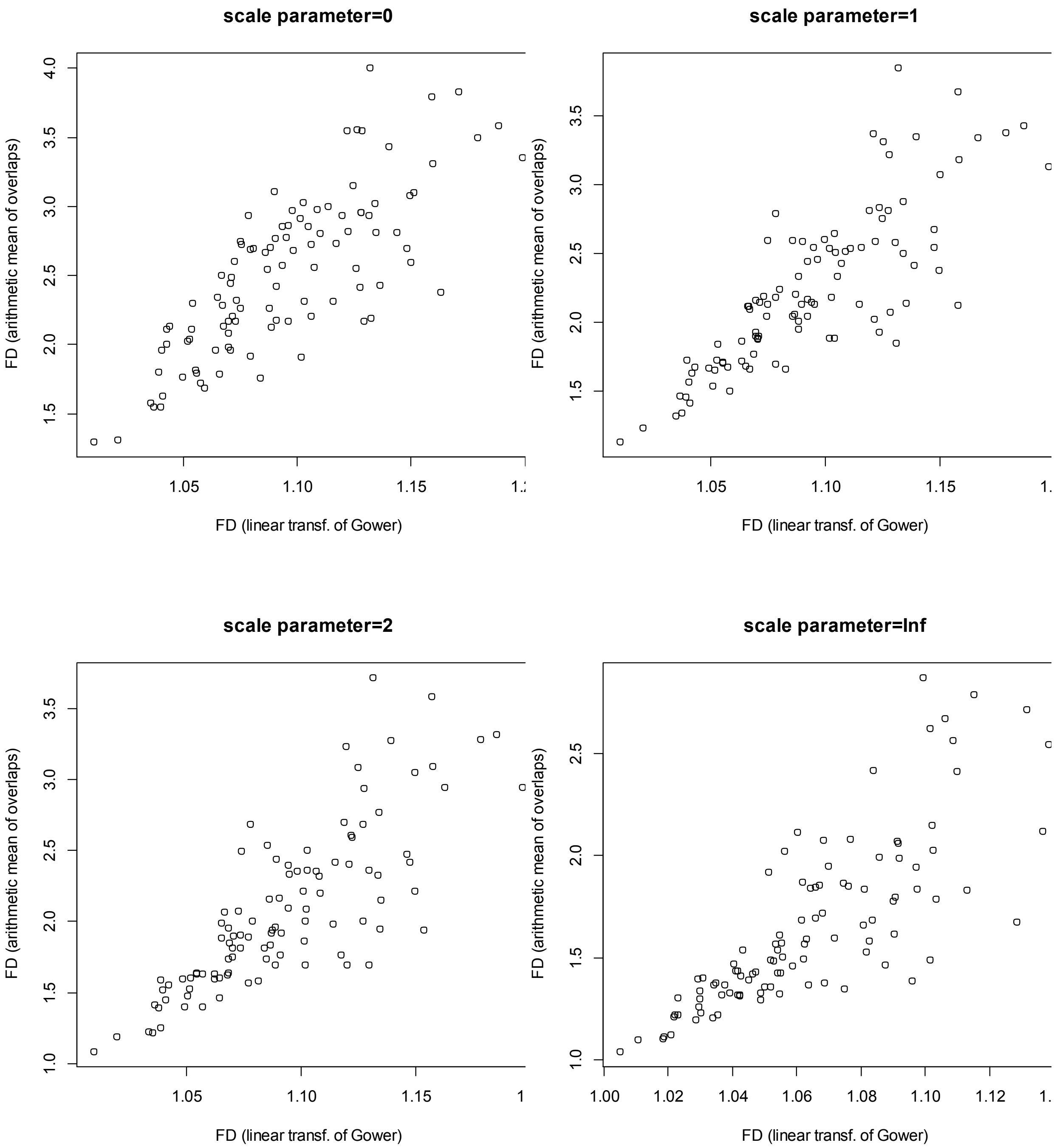

Figure A3: Comparing functional a-diversities calculated from different similarity matrices: linear transformation of Gower-dissimilarity vs arithmetic mean of overlaps

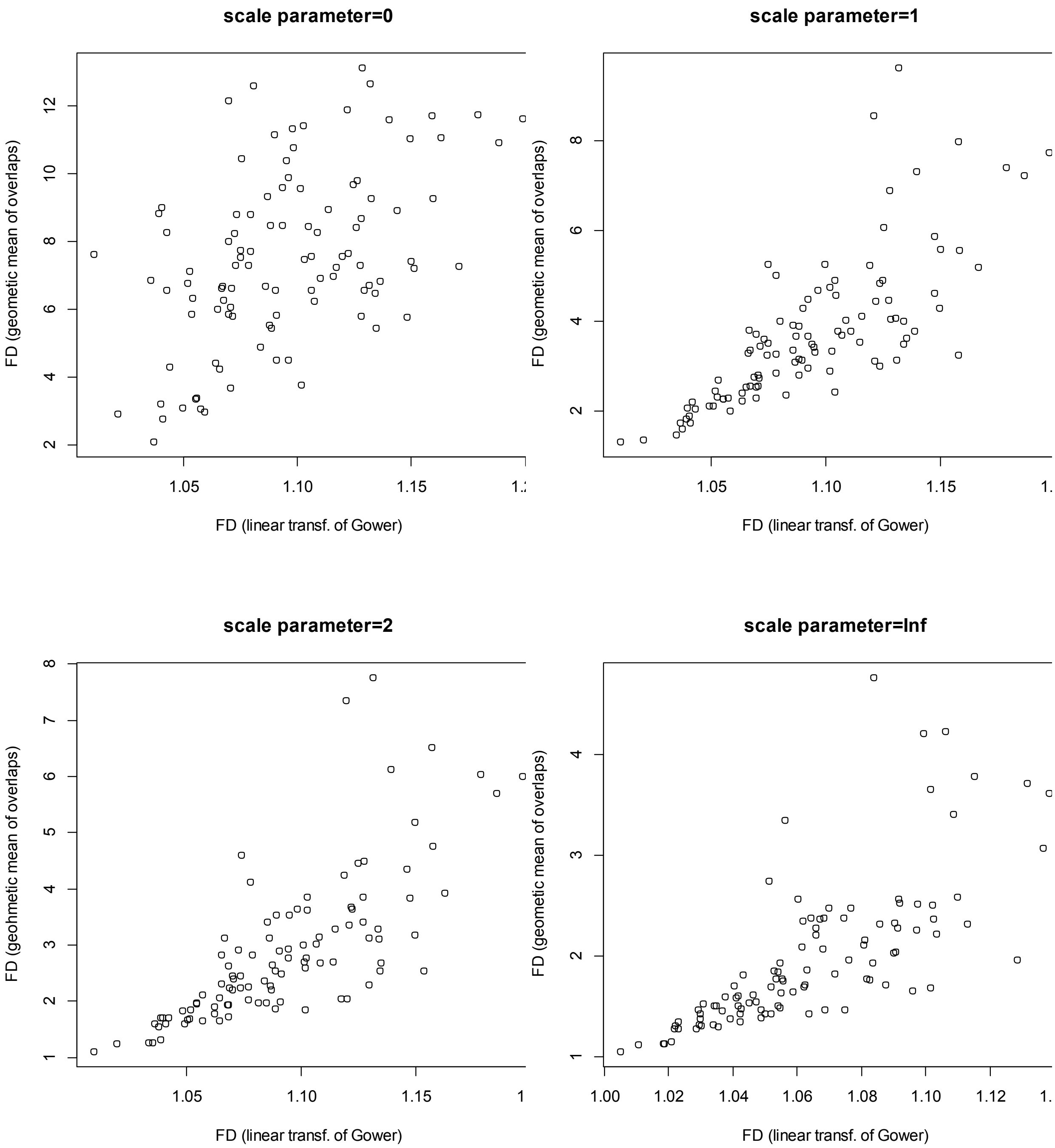

Figure A4: Comparing functional a-diversities calculated from different similarity matrices: linear transformation of Gower-dissimilarity vs geometic mean of overlaps

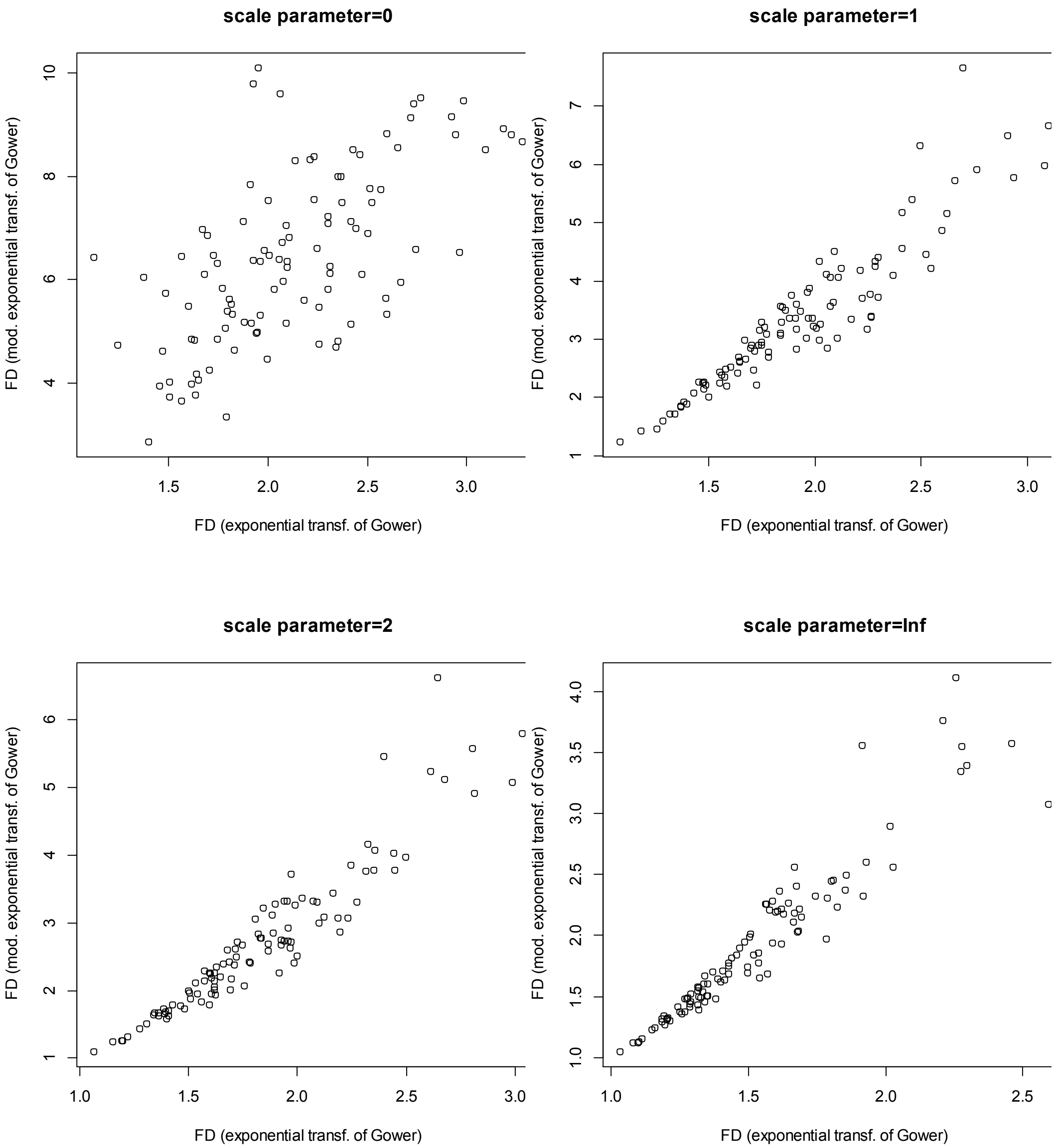

Figure A5: Comparing functional a-diversities calculated from different similarity matrices: exponential vs modified exponential transformation of Gower-dissimilarity

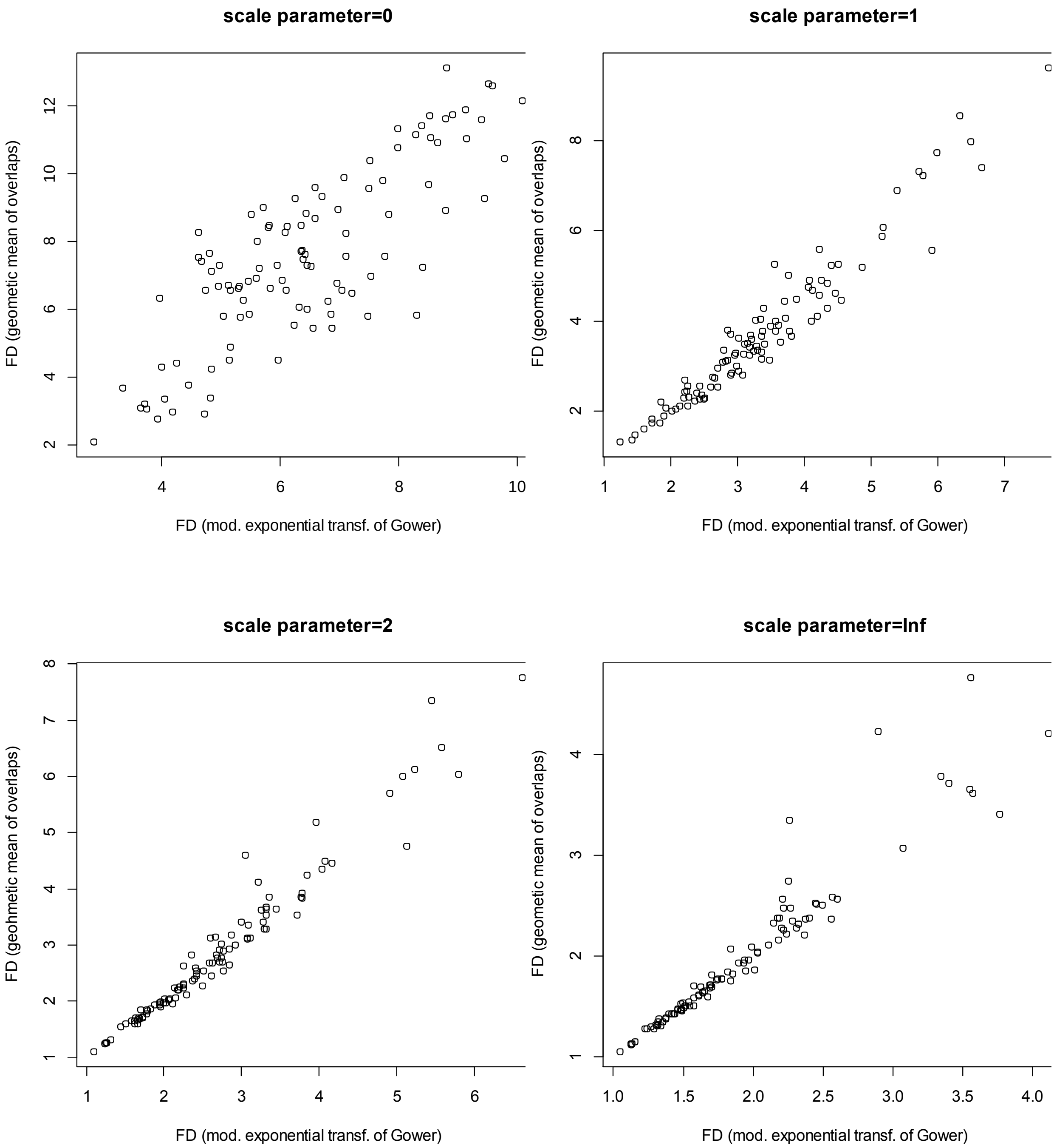

Figure A9: Comparing functional a-diversities calculated from different similarity matrices: modified exponential transformation of Gower-dissimilarity vs geometric mean of overlaps

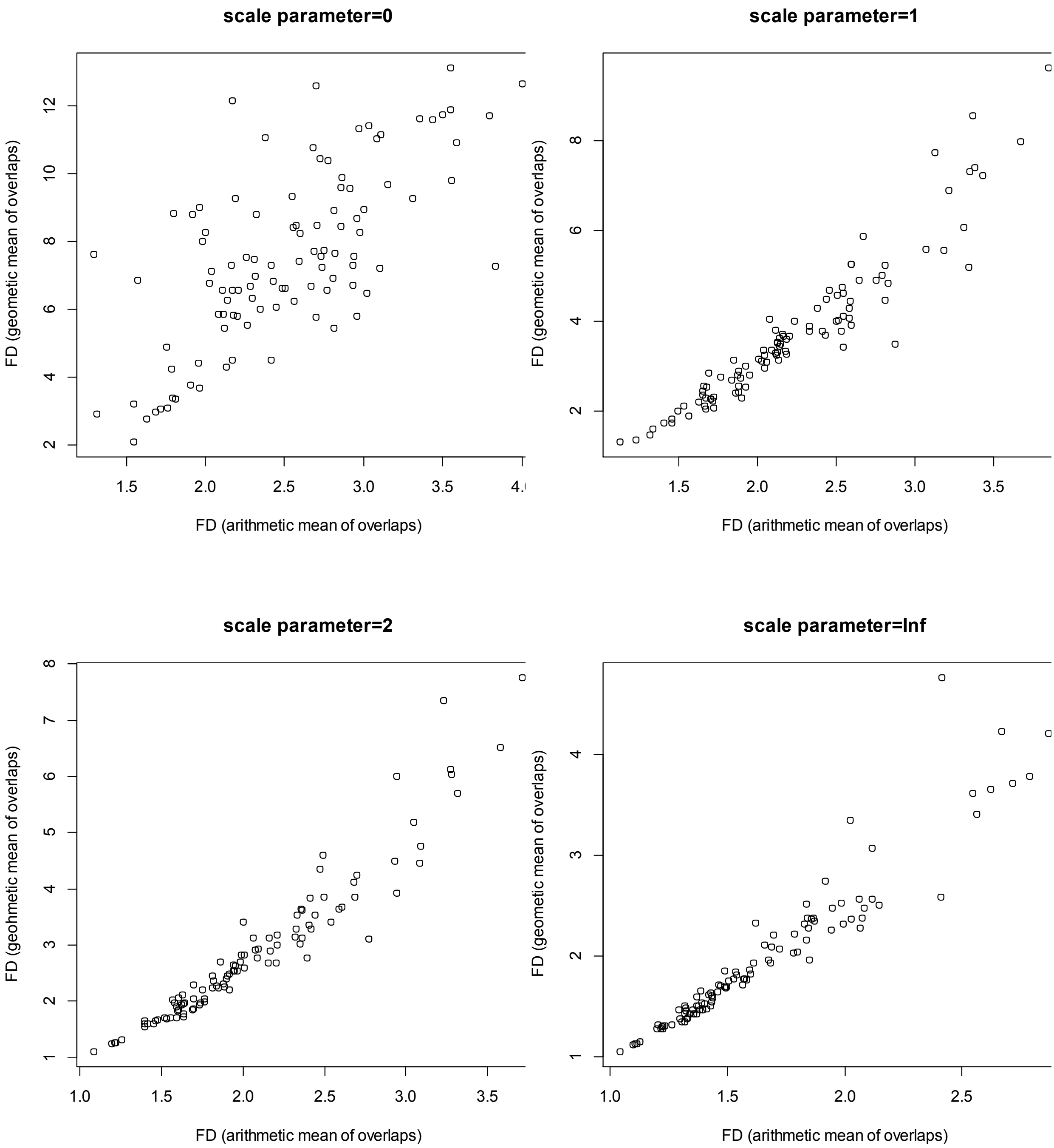

Figure A10: Comparing functional a-diversities calculated from different similarity matrices: arithmetic vs geometric mean of overlaps